

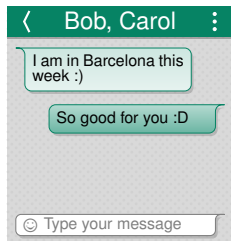
Causal Consistency : Beyond Memory

Matthieu Perrin Achour Mostefaoui Claude Jard

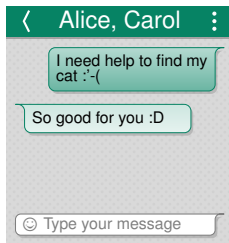
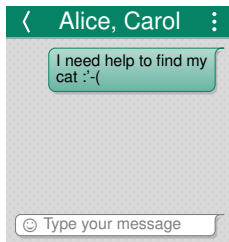
LINA, University of Nantes

March 16th, 2016

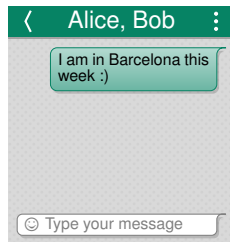
Motivation



At Alice's

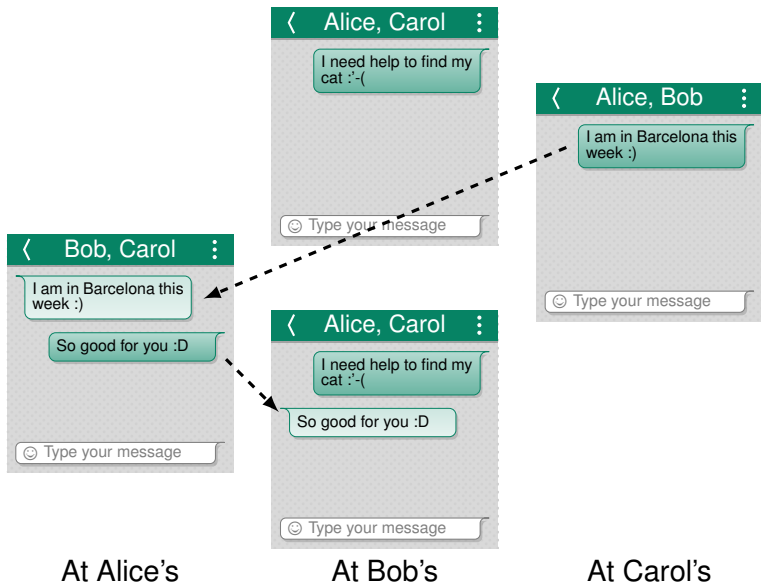


At Bob's



At Carol's


Motivation



What is causal consistency ?

Causal memory¹

- ▶ Build causal order
 - ▶ Writes-into order → semantic dependancies
 - ▶ Process order
- ▶ Each process respects causal order

¹Ahamad et. al. *Causal Memory: Definition, Implementation and Programming*. DISC 1995 


What is causal consistency ?

Causal memory¹

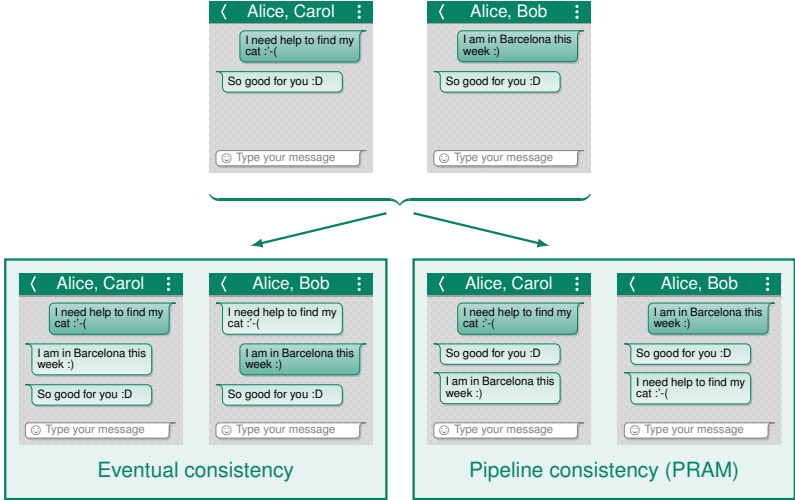
- ▶ Build causal order
 - ▶ Writes-into order → semantic dependancies
 - ▶ Process order
- ▶ Each process respects causal order

Approach

- ▶ Specify data type **sequentially**
- ▶ Suppose existence of a partial order
- ▶ Add links partial order/sequential specification
- ▶ \Rightarrow Semantic dependancies are respected

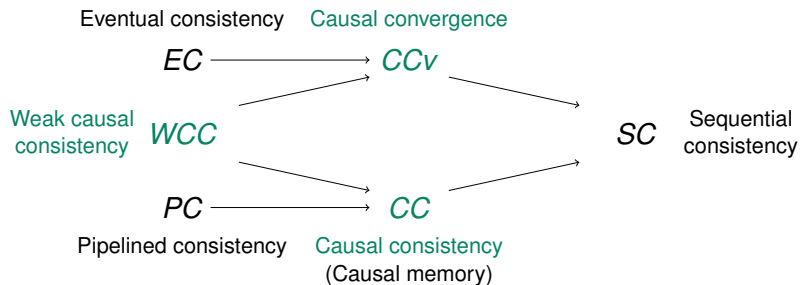
¹Ahamad et. al. *Causal Memory: Definition, Implementation and Programming*. DISC 1995 

Pipeline Consistency versus Eventual Consistency ²



²P. M. J. *Update Consistency in Partitionable Systems*. IPDPS 2015

Contributions



Outline

1. Specifying shared objects
2. Weak causal consistency and causal convergence
3. Causal consistency

Specifying shared objects

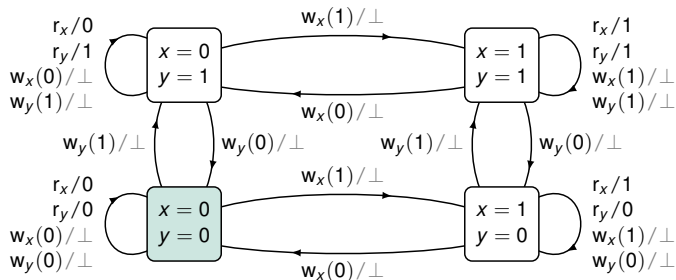
Sequential specification

- ▶ How the object behaves in a sequential environment
 - ▶ Memory
 - ▶ Data base
 - ▶ Instant messaging service

Consistency criterion

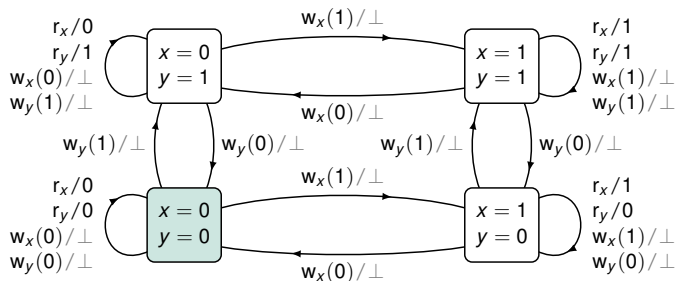
- ▶ How distribution impacts the sequential behaviour
 - ▶ Causal consistency
 - ▶ Eventual consistency
 - ▶ Sequential consistency

Specification of the memory



operation : input/output

Specification of the memory



operation : input/output

Sequential specification

Path in the transition system, starting in q_0

- ▶ finite or infinite
- ▶ hidden operations: $\sigma_i/\sigma_o \Rightarrow \sigma_i$

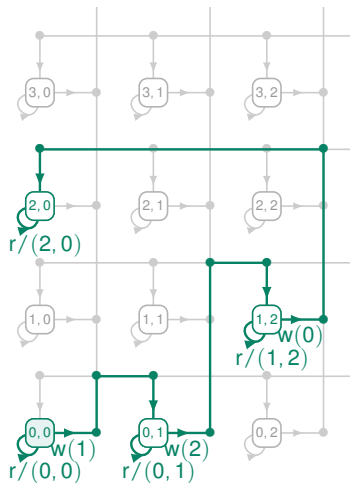
Window stream of size k

Operation $w(n) / \perp$

adds n in the stream

Operation $r / (n_1, \dots, n_k)$

return the k last values



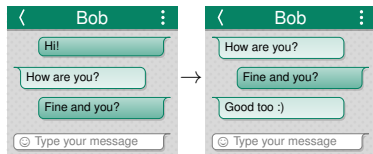
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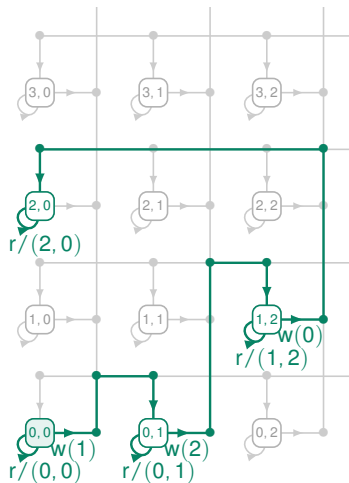
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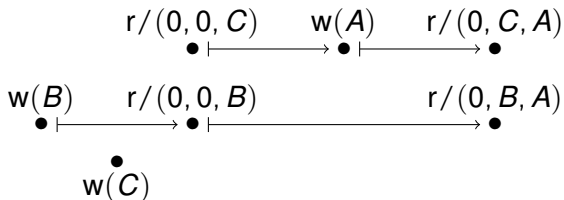
return the k last values



$(1, 2, 3) \xrightarrow{w(4)} (2, 3, 4)$



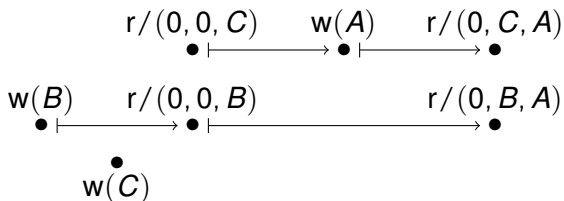
Distributed history



Formally

- ▶ Partially ordered set of events, labelled with operations
- ▶ Each events has a finite past

Distributed history



In systems composed of processes

Events: call of ADT operations

Labels: operations of the ADT

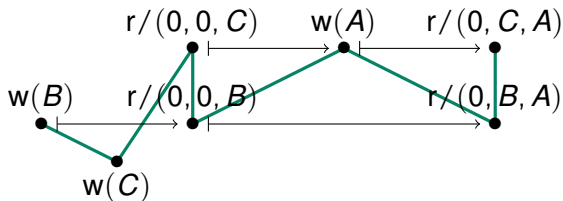
Order: same process, same order

Linearisation

$$I \in \text{lin}(H)$$

- ▶ Sequence of operations
- ▶ Respect of process order

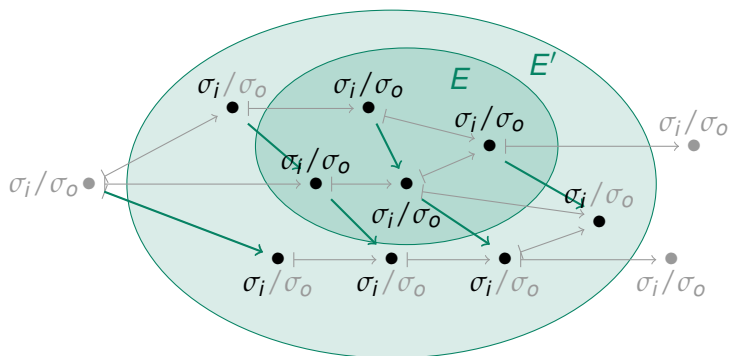
$$I = w(B) \cdot w(C) \cdot r/(0, 0, C) \cdot r/(0, 0, B) \cdot w(A) \cdot r/(0, B, A) \cdot r/(0, C, A)$$



Projection

$$(H^{\rightarrow}).\pi(E, E')$$

- ▶ Replaces process order by \rightarrow
- ▶ Keeps reads of E
- ▶ Keeps writes of E'



Consistency criterion

Sequential specification

- ▶ $L : \text{ADT} \rightarrow \text{set of sequential histories}$

Consistency criterion

- ▶ $C : \text{ADT} \rightarrow \text{set of distributed histories}$

E.g. Sequential Consistency

$$SC : T \mapsto \{H \in \mathbb{H} : \text{lin}(H) \cap L(T) \neq \emptyset\}$$

Outline

1. Specifying shared objects
- 2. Weak causal consistency and causal convergence**
3. Causal consistency

Causal order and time zones

Causal order →

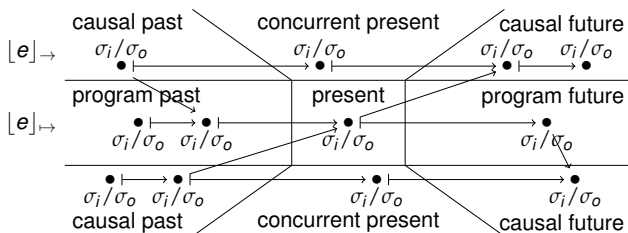
- ▶ Partial order on all events
- ▶ Contains process order
- ▶ For each event : **cofinite** causal future
 - ▶ necessary to make it not trivial
 - ▶ necessary for pipelined consistency $PC \leq CC$
 - ▶ necessary for eventual consistency $EC \leq CC_v$

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6 time zones per event

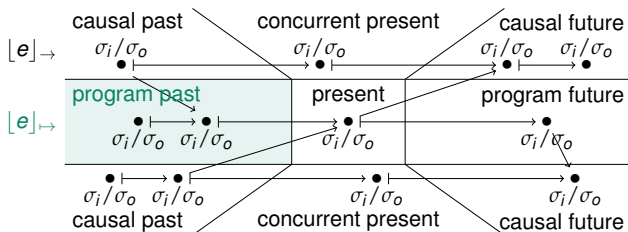


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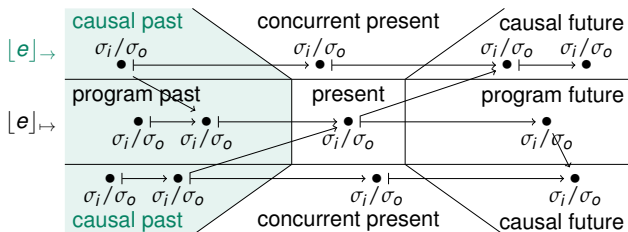


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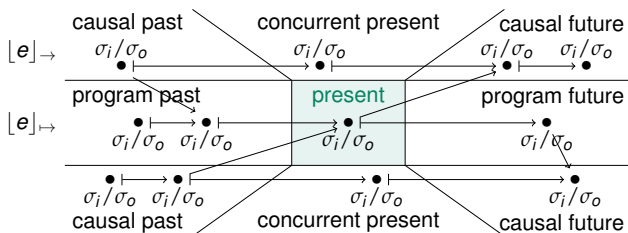


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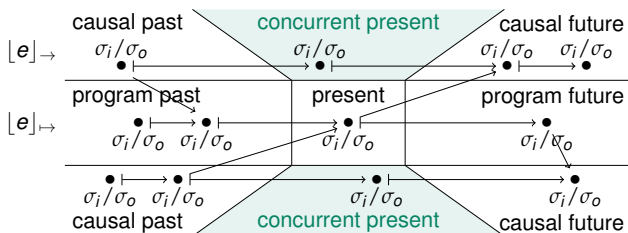


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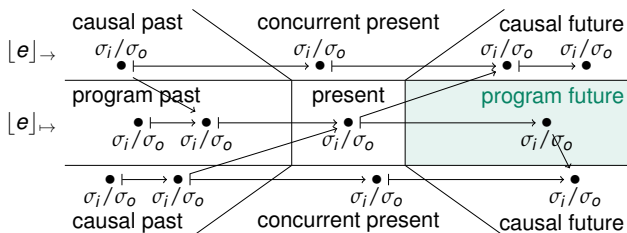


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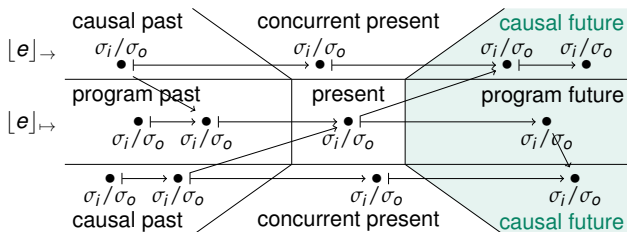


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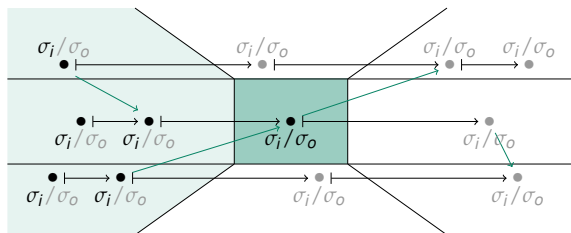


Weak causal consistency

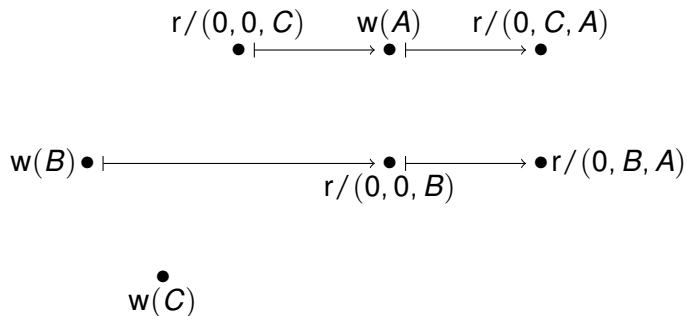
$H \in WCC(T)$ if

- ▶ there exists a causal order \rightarrow , for each event e

$$\text{lin}((H^{\rightarrow}).\pi(\{e\}, [e]_{\rightarrow})) \cap L(T) \neq \emptyset$$

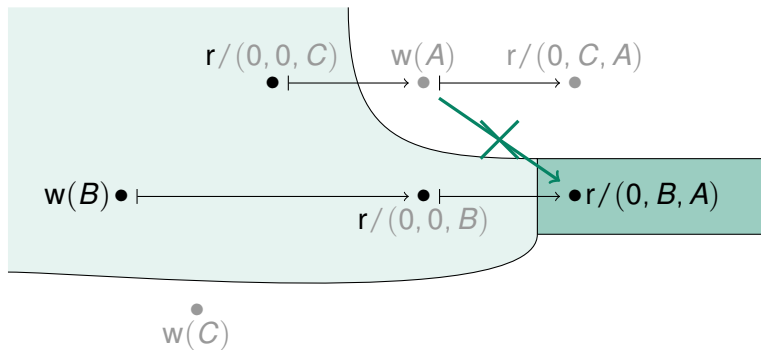


Alice, Bob and Carol



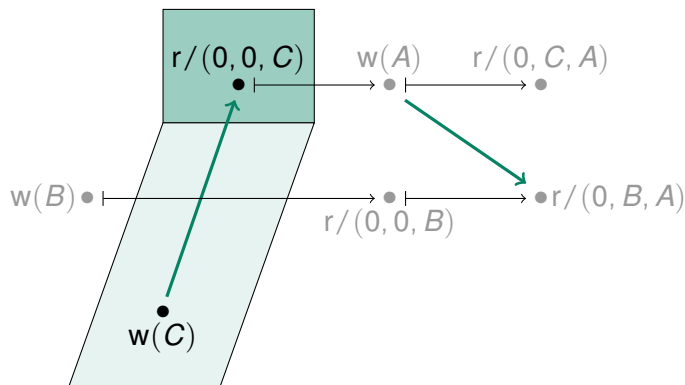
$H \in WCC(\mathcal{W}_3)$?

Alice, Bob and Carol



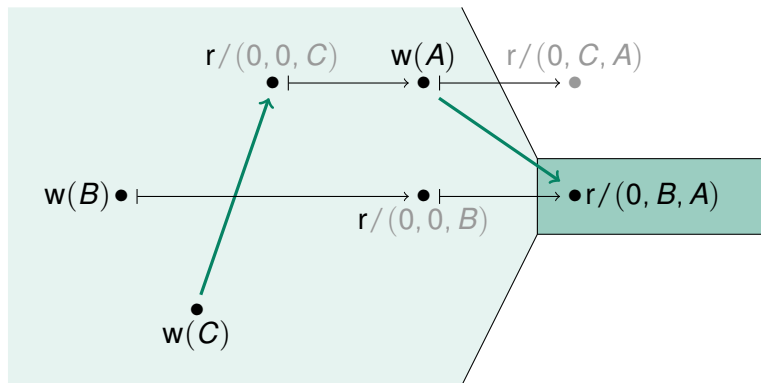
$H \in WCC(\mathcal{W}_3)$?

Alice, Bob and Carol



$H \in WCC(\mathcal{W}_3)$?

Alice, Bob and Carol



$H \in WCC(\mathcal{W}_3)$?

Causal convergence

$H \in CCv(T)$ if

- ▶ there exists
 - ▶ a causal order \rightarrow
 - ▶ a total order \leq that contains \rightarrow
- ▶ for each event e

$$\text{lin}((H^{\leq}).\pi(\{e\}, [e]_{\rightarrow})) \cap L(T) \neq \emptyset$$

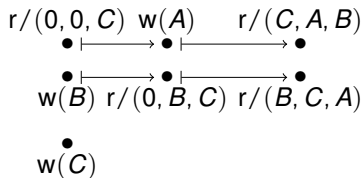
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Examples



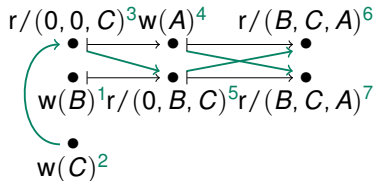
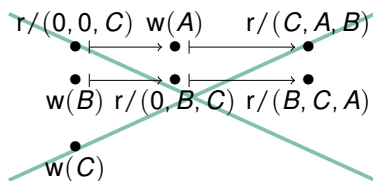
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Examples



Properties

- ▶ H : distributed history
- ▶ T : abstract data type
- ▶ write : not a loop
- ▶ read : output depends of the state

Weak causal consistency

$$\begin{aligned} & H \in WCC(T) \\ \wedge & \text{ no concurrent writes} \\ \implies & H \in SC(T) \end{aligned}$$

Causal convergence

$$\begin{aligned} & H \in CCv(T) \\ \wedge & \text{ no read concurrent with a write} \\ \implies & H \in SC(T) \end{aligned}$$

Outline

1. Specifying shared objects
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Causal memory

Writes-into order

- ▶ Relation $w_x(n) \dashv\circ r_x/n$
- ▶ At most one predecessor per read
- ▶ Read 0 if no predecessor

Causal memory

There exists $\dashv\circ$ such that

- ▶ there exists a partial order \rightarrow that contains $\dashv\circ$ and \mapsto
- ▶ for each process p , $\text{lin}((H^\rightarrow).\pi(p, E_H)) \cap L(\mathcal{M}) \neq \emptyset$

Pipeline Consistency³

$H \in PC(T)$ if

- ▶ For each process p

$$\text{lin}((H^{\rightarrow}) \cdot \pi(p, E)) \cap L(T) \neq \emptyset$$

³Lipton, Sandberg. *PRAM: A Scalable Shared Memory*. 1988

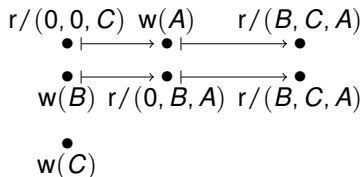
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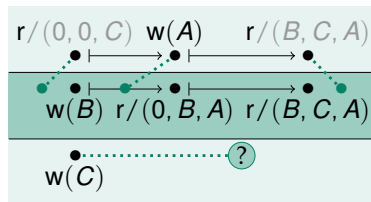
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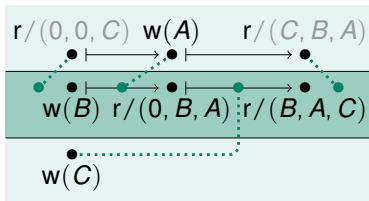
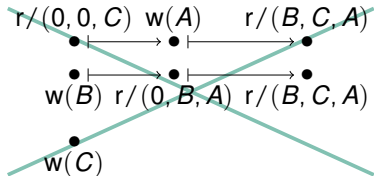
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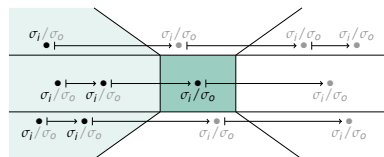
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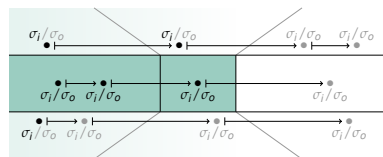
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Causal consistency

Weak causal consistency

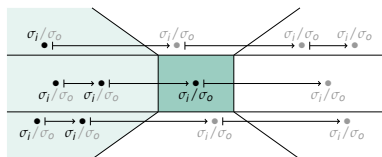


Pipeline consistency

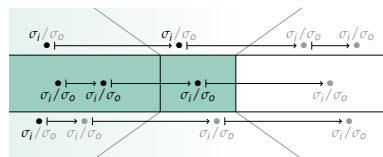


Causal consistency

Weak causal consistency



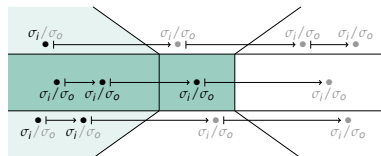
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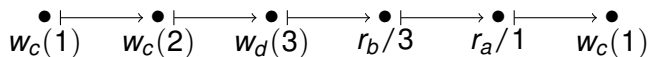
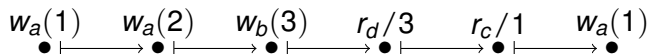
Causal consistency: $H \in CC(T)$ if

- ▶ There exists a causal order \rightarrow , for each event e

$$\text{lin}((H^{\rightarrow}).\pi([e]_{\mapsto}, [e]_{\rightarrow})) \cap L(T) \neq \emptyset$$



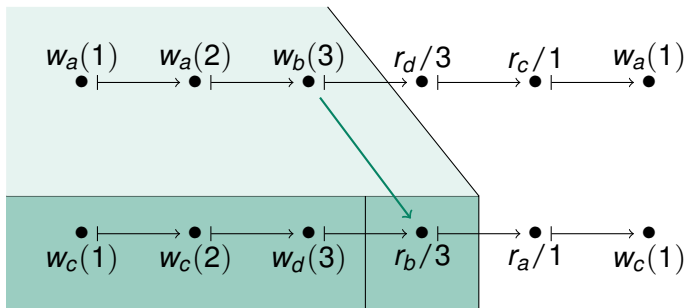
Limit of causal memory



Causal consistency?

Causal memory?

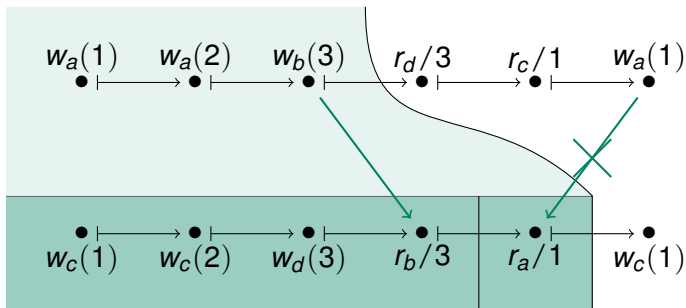
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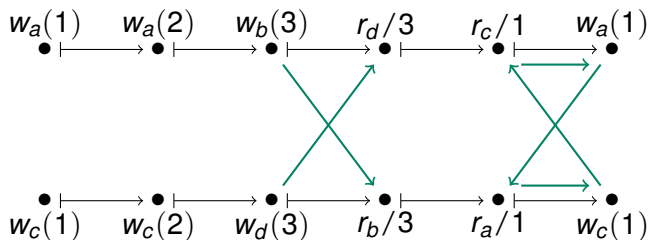
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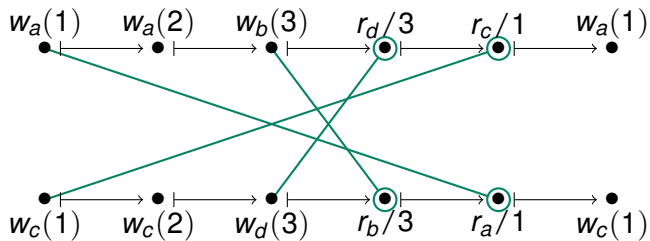
Limit of causal memory



~~Causal consistency~~

Causal memory?

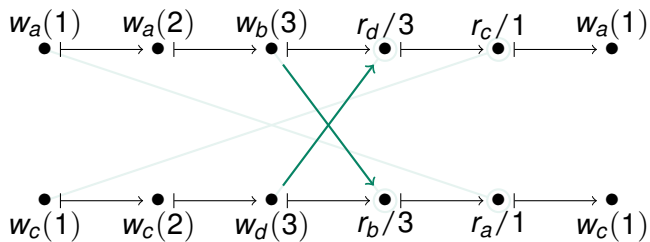
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~~Causal consistency~~

Causal memory?

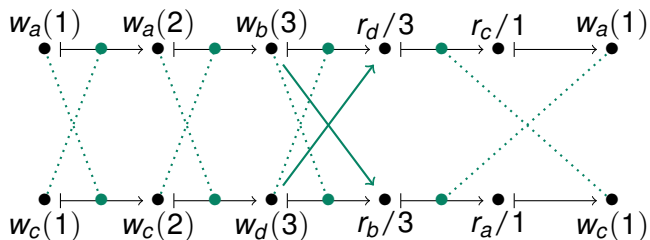
Limit of causal memory



~~Causal consistency~~

Causal memory?

Limit of causal memory



~~Causal consistency~~

Causal memory?

Link with causal memory

Property 1

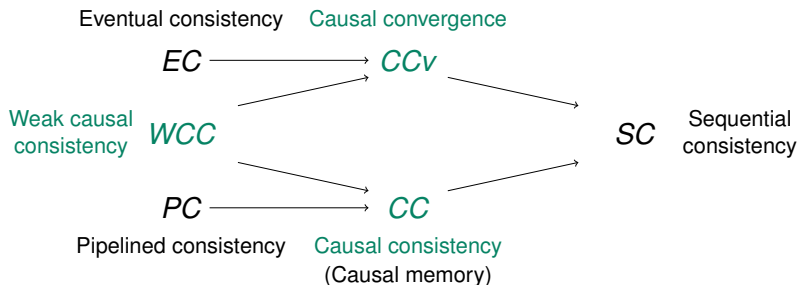
H causally consistent for memory
 $\implies H$ consistent for causal memory

Property 2

Suppose H does not contain twice $w_x(n)$.

H consistent for causal memory
 $\implies H$ causally consistent for memory

Conclusion



When to use?

Causal convergence: for distributed systems

Causal consistency: for parallel algorithms